Experiment HH-4: The Six-Lead Electrocardiogram

Appendix: Equations Used by LabScribe to Graph the Augmented Limb Leads

The ECG is a representation of the electrical activity of the heart. It is a vector quantity with magnitude and direction. The various leads (I, II, III, aVL, aVR, aVF) measure the magnitude of the ECG along a particular direction (Figure HH-4-A1).

The derivations of the equations used to calculate the cardiograms for the four other limb leads (III, aVL, aVR, aVF) from the cardiograms recorded from Leads I and II are provided for your information. These equations do not need to be used to perform this experiment. The computed functions that calculate the four remaining limb leads are already programmed in the settings file for this experiment.
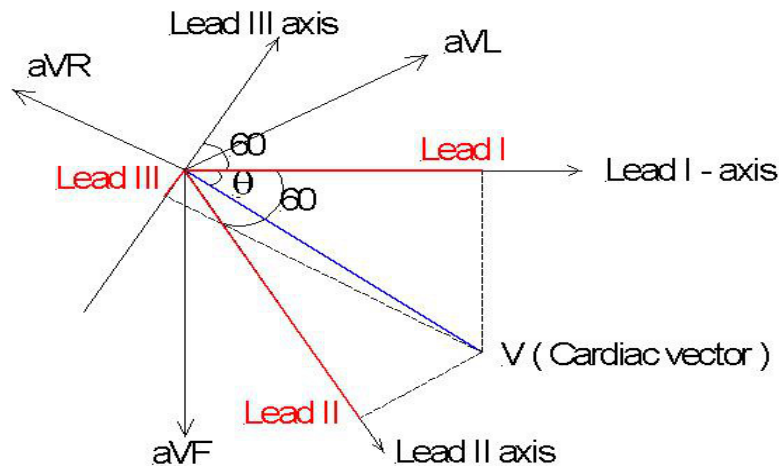

Figure HH-4-A1: Lead I is the projection of the cardiac vector (V) along the Lead I axis. Similarly Lead II is the projection of cardiac vector along the Lead II axis, and so on.

Treat ECG as a Vector with magnitude V and an angle θ:

$$\text{Lead I} = V\cos(\theta)$$

$$\text{Lead II} = V\cos(60 - \theta)$$

$$\text{Lead III} = -V\cos(60 + \theta)$$

$$aVF = V\sin(\theta)$$

$$aVL = V\cos(30 + \theta)$$

$$aVR = -V\cos(30 - \theta)$$

1. Calculate Lead III from Leads I and II:

$$\text{Lead I} = I = V\cos(\theta) \tag{1}$$

$$\text{Lead II} = II = V\cos(60 - \theta) \tag{2}$$

$$\text{Lead III} = III = -V\cos(60 + \theta) \tag{3}$$

   Using the following trigonometric identities:

$$\cos(A + B) = \cos A \cos B - \sin A \sin B \tag{4}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B \tag{5}$$

2. From equations 3 and 4:

$$III = -V\{\cos(60)*\cos(\theta) - \sin(60)*\sin(\theta)\} \tag{6}$$

3. Using equations 2 and 5:

$$II = V\{\cos(60)*\cos(\theta) + \sin(60)*\sin(\theta)\} \tag{7}$$

4. Adding equations 6 and 7:

$$II - III = 2*V*\cos(60)*\cos(\theta) = 2*V*0.5*\cos(\theta) = V\cos(\theta)$$

5. Therefore:

$$II - III = I; III = II - I; \text{ and, Lead III} = \text{Lead II} - \text{Lead I}$$

Calculate aVF from Leads I and II:

$$aVF = V\sin(\theta) \tag{8}$$

1. From equation 7:

$$II = V\{\cos(60)*\cos(\theta) + \sin(60)*\sin(\theta)\}$$

$$II = (1/2)*V\cos(\theta) + (\surd3)/2*V*\sin(\theta)$$

$$2(II) = I+(\surd3)*V*\sin(\theta)$$

2. Therefore:

$$V*\sin(\theta) = (2(II) - I)/ \surd3 \tag{9}$$

   and:

$$aVF = (2(II) - I)/ \surd3 \tag{10}$$

Calculating aVL from Leads I and II:

$$aVL = Vcos(30+\theta)$$

1. Therefore:

$$aVL = V\{cos(30)*cos(\theta) - sin(30)*sin(\theta)\}$$

$$aVL = cos(30)*Vcos(\theta) - sin(3)*Vsin(\theta)$$

$$aVL = cos(30)*I - sin(30)*aVF$$

2. Therefore:

$$aVL = ((\sqrt{3})/2)*I - (1/2)*(2*II - I)/\sqrt{3}$$

$$aVL = ((\sqrt{3})/2)*I - II/\sqrt{3} + (1/2)*I/\sqrt{3}$$

$$aVL = (2*I - II)/\sqrt{3} \qquad\qquad (11)$$

Calculate aVR from Leads I and II:

$$aVR = -Vcos(30-\theta)$$

1. Therefore:

$$aVR = -V\{cos(30)*cos(\theta) + sin(30)*sin(\theta)\}$$

$$aVR = -cos(30)*I - sin(30)*aVF$$

2. Therefore:

$$aVR = -((\sqrt{3})/2)*I - (1/2)*(2*II - I)/\sqrt{3}$$

$$aVR = -((\sqrt{3})/2)*I - II/\sqrt{3} + (1/2)*I/\sqrt{3}$$

$$aVR = -(I+ II)/\sqrt{3} \qquad\qquad (12)$$

Calculate Cardiac Angle from Leads I and II:

1. From equation 9:

$$V*sin(\theta) = (2*II - I)/\sqrt{3}$$

2. From equation 1:

$$Vcos(\theta) = I$$

3. Dividing equation 9 by equation 1:

$$tan(\theta) = \{(2*II - I)/\sqrt{3}\}/I$$

$$\theta = + Arctan((2*II - I)/\sqrt{3} *I) \qquad\qquad (13)$$